The Open University

Science
S382 Astrophysics

S382

Tutor-Marked Assignment 01

Contents **Cut-off date**

Please send all your answers to the tutor-marked assignment (TMA) to reach your tutor by 12 noon (UK local time) on or before the cut-off date shown on the S382 website. Your TMAs should be submitted through the eTMA system unless there are difficulties which prevent you from doing so. In these circumstances, you must negotiate with your tutor to get their agreement to submit your assignment on paper. The eTMA system allows for eTMA submission directly to the university 24 hours a day, and either gives you confirmation that your eTMA has been submitted successfully or, if there has been a problem, an error message informing you of the problem and what steps you can take to overcome it. If you submit online you must keep your receipt code in case you need to prove successful submission.

General information about policy and procedure is in the *Assessment Handbook* which you can access from StudentHome. However, there are a number of ways in which S382 eTMA submission differs from what is described there. These are described in the document *Producing eTMAs for Level 3 physics and astronomy modules* on the S382 website. See also the S382 *Introduction and Guide* for module-specific information.

Of particular importance is the test submission, TMA 00. This will enable you to familiarize yourself with the system and allow your tutor to check that the format in which you save your TMAs is compatible with their own computer software. It is your responsibility to make sure that you submit documents in a compatible format and we strongly recommend that you submit TMA 00. TMAs submitted in an incorrect format may not be marked.

If you are submitting a paper copy, please allow sufficient time in the post for the assignment to reach its destination on or before the cut-off date. We strongly advise you to use first-class post and to ask for proof of postage. Do not use recorded delivery or registered post as your tutor may not be in to receive it. Keep a copy of the assignment in case it goes astray in the post. You should also include an appropriately completed assignment form (PT3). You will find instructions on how to fill in the PT3 form in the *Assessment Handbook*. Remember to fill in the correct assignment number (01).

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2.1 PT/TeX

This first booklet provides general advice on answering TMA questions as well as the questions for TMA 01. Although the marks for your assignments do not count directly towards your S382 result, they are an essential part of your learning and you are required to engage satisfactorily with them. Please refer to the S382 *Introduction and Guide* for additional information about the module assessment.

Assignment cut-off dates

The cut-off dates for the assignments provide an important element of pacing for your study of S382 and they are spread fairly uniformly through the year, leading up to the exam. **You should regard these dates as fixed points**. *Any extension to a TMA cut-off date requires prior permission from your tutor, which may not always be given. Extensions may be granted in exceptional circumstances but it will never be possible to have an extension of more than 3 weeks.* Your tutor will, of course, be willing to discuss with you the best strategies for catching up if you have fallen behind, and should be able to help with questions if you are stuck.

Marking of assignments

As explained in the *Introduction and Guide*, all the assignments for S382 are *formative*. They are designed to help with the teaching of the module not its assessment, and the scores you obtain for them *do not* count towards your overall grade. Nonetheless, **you are required to satisfactorily complete at least 8 out of 10 of the TMAs and iCMAs** in order to be considered for a grade based on the examinable assessment components (i.e. the exam covering Parts 1 and 3 of the module and the project portfolio covering Part 2 of the module).

The assignment questions allow you to demonstrate that you have achieved particular learning outcomes for the module. These learning outcomes are listed in Section 2 of the *Introduction and Guide*. They include knowledge and understanding of the module content, the ability to apply this knowledge and understanding to the solution of problems in astrophysics, the ability to explain concepts, phenomena and applications in astrophysics, and the ability to communicate effectively your solutions and explanations. In each assignment booklet we indicate which of the learning outcomes are assessed in the assignment, and which parts of the questions relate to which learning outcomes.

When commenting on your assignment answers, your tutor will be assessing the extent to which you have achieved the learning outcomes. This will include assessing whether you have got the correct answer, but also whether you have explained your reasoning, whether your answers are well-structured (both for numerical and discursive answers), and whether you have used correct terminology and notation, and so on. For each of the learning outcomes that are assessed, your tutor will allocate a descriptor to indicate your level of achievement, which will be one of: well demonstrated, demonstrated, just demonstrated, not quite demonstrated, not demonstrated (or not attempted).

General advice on answering S382 assignment questions

We summarize below a number of general points that you should have in mind when answering the TMA questions. Many of these are likely to be familiar from previous modules, but they bear repetition at the start of a new module.

1. **Define the terms used** Good scientific communication requires clarity about the meaning of terms and symbols. You will be expected to state the meaning of scientific terms that you use in your answers if they are not introduced in the question.
2. **Appropriate use of notation** A great deal of meaning is conveyed by notation. You have freedom to choose symbols for a given situation, but should avoid potentially misleading choices. Using the same notation as used in the printed books is usually a good idea.
3. **Laying out answers to problems** Effective communication with your tutor (or anyone else) is helped by a neat and logical layout of your answers. Perhaps the best advice is for you to look carefully at the layout of the solutions to worked examples and exercises in the S382 books. These solutions also provide examples of the appropriate level of explanation required in your TMA answers. Note that, although you may be able

to solve a particular problem by writing out a few lines of algebra that lead to the correct answer, you will only be awarded full credit if you provide explanation and justification at appropriate stages in your answer.

In answering many questions you will have to write down and manipulate algebraic expressions. If you are not confident or proficient at doing equation typesetting within a word processor package, but still want to submit your answers using the eTMA system, please write your answers by hand and then scan or photograph them in order to submit them electronically. We would far prefer that you spent your time concentrating on the astrophysics than the typesetting!

4. **Answers requiring written explanation or description** Some of the TMA questions allow you to demonstrate and develop your scientific writing skills. Your tutor will be assessing whether your answer is coherent (i.e. topics and argument presented in a logical order and clearly linked to the question), clear (i.e. the meaning is unambiguous, and there is correct use of English and appropriate equations), and concise. Guidance on the length of written answers will generally be given. If you are tempted to write more than the guideline indicates, it may be because you have not written a clear and concise answer, or you may have included material that is irrelevant or secondary, and you could lose marks as a result. Conversely, if your answer is much shorter than the guideline indicates, then you should consider whether you have omitted relevant information, or written so tersely as to make the answer difficult to follow.
5. **Include equations and diagrams where helpful** When answering a discursive question, do not hesitate to quote relevant equations. In astrophysics, equations are often the most precise and succinct way of presenting information. Remember to define in words the meanings of any symbols that may not be obvious. Hand-sketched diagrams can also help to clarify an answer, and can save many words of description. Simple pencil-drawn line diagrams are all that is generally required — neat, but not necessarily artistic. If submitting electronically, you may wish to scan in or photograph a hand-drawn sketch.
6. **Units** The values of most physical quantities have units as well as a numerical value. For a value of energy E, it would be correct to write '$E = 2.5\,\mathrm{J}$', but it certainly would not be acceptable to write '$E = 2.5$'. Also, it is not correct to write 'E joules', since we assume that a symbol such as E, representing an energy, already contains a unit for energy. Mathematicians sometimes take liberties about how they handle units, but scientists generally treat units much more seriously. Answers to problems should *always* include the appropriate unit, and when writing out equations that include numerical values, the units on each side of the equation must always balance. The solutions to the exercises in the S382 books provide many examples of good practice.
7. **Significant figures** Numerical answers should always be quoted to the appropriate number of significant figures. In this module we often quote values for the quantities used in numerical calculations to two significant figures, and we provide values of physical constants to three significant figures. In such cases, the answers should be quoted to two significant figures. For intermediate steps in a calculation it is good practice to record *one more* figure than is significant, to avoid the accumulation of rounding errors in the intermediate steps, leading to an error in the final answer.
8. **Checking answers** We all make slips in algebraic derivations and in evaluating numerical results, so it is advisable to check your answers. There are various ways to do this, depending on the problem. These include checking that units balance in derived equations, checking numerical results with rough estimates obtained by rounding all of the numbers in the calculation, looking at limiting cases for algebraic expressions and asking yourself whether the limiting behaviour is reasonable, checking whether an expression increases or decreases in an appropriate way when the value of a parameter is changed, and so on.
9. **Plagiarism** You are encouraged to discuss the S382 materials and assignment questions with other students, but the answers to the assignment questions must be your own work. This does not preclude you from making judicious use of material from other sources, but you must acknowledge such use by giving the author's name, the year of

publication, the name of the publication in which it appears (or the website address), and the edition or volume number and the page number. However, there is no need to give references for standard equations in the S382 texts. You are advised to read the University's guidelines on plagiarism, see the *Introduction and Guide* for more details.

To check that all students are working in a fair and academically appropriate manner, the Open University is currently using some text-comparison software to detect potential cases of plagiarism in work that is submitted for assessment. Details of how this is implemented in this module are given on the S382 website.

TMA 01   CUT-OFF DATE: SEE S382 STUDY PLANNER

This assignment covers Chapters 3 and 4 of *Stellar Evolution and Nucleosynthesis*. It allows you to assess your ability to achieve the following learning outcomes:

Kn2: Knowledge and understanding of basic concepts of quantum physics, nuclear physics and particle physics that are of relevance to astrophysics and cosmology.
Kn4: Knowledge and understanding of the physical processes that sustain the energy output of stars during each stage of their evolution and drive the progression from one stage to the next.
C1: Manipulate numbers, algebraic symbols and mathematical functions in equations of relevance to astrophysics and cosmology.
C3: Derive and manipulate quantitative theoretical models of physical processes and derive physical estimates.

Question 1

This question relates to the physics of stars on the main sequence and the nuclear fusion reactions that occur within their cores. Parts (a)–(f) relate to the proton–proton chain as discussed in Chapter 3, whilst part (g) uses the homology argument developed in Chapter 4.

(a) (**Learning outcome C1**)
The second step of the proton–proton chain in the Sun is the reaction:

$$\mathrm{p} + \mathrm{d} \longrightarrow {}^{3}_{2}\mathrm{He} + \gamma.$$

Show that the reduced mass of a system consisting of one deuteron and one proton may be expressed:

(i) as 2/3 in atomic mass units and (ii) as 1.11×10^{-27} kg.

(For simplicity, you may assume that the mass of the deuteron is twice the mass of the proton, and that the mass of the proton can be approximated by the atomic mass unit, $m_\mathrm{p} = u = 1.661 \times 10^{-27}$ kg.)

(b) (**Learning outcomes Kn2 & C1**)
Show that the Gamow energy for the fusion of a deuteron and a proton is 655 keV.

(*Hint:* Use the reduced mass you have calculated in (a).)

(c) (**Learning outcomes Kn2 & C1**)
Using the reduced mass from (a), the Gamow energy from (b) and the data for the properties of the Sun's core below, show that the reaction rate R_pd for the fusion of a proton and a deuteron in the core of the Sun is comparable to the reaction rate for the fusion of two protons, calculated in Section 3.3 of the book.

(You will probably find it useful to begin by evaluating kT in units of keV. You may assume that in the Sun's core: core temperature $T_\mathrm{c} = 15.6 \times 10^6$ K, proton number density $n_\mathrm{p} = 4.42 \times 10^{31}\ \mathrm{m}^{-3}$, deuteron number density $n_\mathrm{d} = 1.44 \times 10^{14}\ \mathrm{m}^{-3}$, S-factor at the Gamow peak $S(E_0) = 2.5 \times 10^{-4}$ keV barns.)

(d) (**Learning outcomes Kn2 & C1**)
Show that the mean lifetime of a deuteron in the core of the Sun is about 1 second. (Use your result from (c).)

(e) (**Learning outcome Kn4**)
(i) How does the mean lifetime of a deuteron in the core of the Sun compare with the mean lifetime of a proton in the core of the Sun?

(ii) By considering the cross-sections for the two reaction steps, explain why one of these two steps determines the overall rate of reaction.

(iii) By equating the rates of the two reaction steps, and expressing them in terms of the particle number densities and reaction cross-sections, comment on what determines the ratio of the density of deuterons to the density of protons in the Sun's core.

(f) (**Learning outcomes Kn2 & C3**)
By considering their functional dependences, explain how:

(i) the energy of the Gamow peak, E_0,

(ii) the Gamow width, Δ, and

(iii) the temperature exponent (power), ν, of the proton + deuteron fusion reaction

compare with those of the first stage of the proton–proton chain. (You do *not* need to calculate any of these quantities.)

(g) (**Learning outcome C3**)
(i) Combine the energy generation equation (Equation 1.15 – one of the four equations of stellar structure) with the energy generation rate equation (Equation 3.33) for the proton–proton chain (assuming the temperature exponent $\nu \sim 4$), to obtain the rate of change of luminosity with radius as a function of radius, density and temperature.

(ii) Use a homology argument, similar to that in Section 4.3, to write a scaling relation for this energy generation equation, i.e. luminosity as a function of radius, density and core temperature.

(iii) Substitute scaling relations for density and core temperature as a function of mass and radius into your result from (ii) (Equations 4.2 and 4.5, assuming a constant chemical composition), to obtain a scaling relation for luminosity as a function of mass and radius only.

(iv) Combine your result from (iii) with the Eddington mass–luminosity–radius relationship (Equation 1.18) to obtain a scaling relationship between luminosity and radius *only*.

(v) Finally, write a scaling relationship for the Stefan–Boltzmann equation (Equation 1.1) and combine that with your result from (iv) to show that the form of the lower main sequence on the Hertzsprung–Russell diagram may be expressed as

$$\log_{10} L \sim \frac{284}{69} \log_{10} T_{\text{eff}} + \text{constant}.$$

(vi) What assumptions about the properties of stars have gone into this derivation?

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